Magnetic Field

Constants

Permeability of free space: $\mu_0 = 4\pi \times 10^{-7} \mathrm{N/A} = 1.26 \times 10^{-6} \mathrm{N/A}$

$$c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Biot-Savart Law

$$\mathrm{d}B = \frac{\mu_0 I \,\mathrm{d}l}{4\pi r^2} \sin\theta$$

$$\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \int \frac{I}{r^2} \,\mathrm{d}\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}$$

Finite current wire

$$B = \frac{\mu_0}{4\pi} \frac{I}{d} \int_{\theta_1}^{\theta_2} \sin\theta \,\mathrm{d}\theta = \frac{\mu_0}{4\pi} \frac{I}{d} (\cos\theta_1 - \cos\theta_2)$$

To infinity: $B = \dfrac{\mu_0 I}{2\pi d}$.

Finite plate

$$\begin{aligned} B &= \int_{-L/2}^{L/2} \mathrm{d}\left(B \frac{d}{r}\right) = \int_{-L/2}^{L/2} \frac{\mu_0}{2\pi r} \frac{I}{L} \frac{d}{r} \,\mathrm{d}x \\ &= \int_{-L/2}^{L/2} \frac{\mu_0 I d}{2\pi L} \frac{\mathrm{d}x}{x^2 + d^2} = \frac{\mu_0 I}{\pi L} \arctan \frac{L}{2d} \end{aligned}$$

$$\begin{aligned} d \gg L \quad &\Rightarrow \quad B = \frac{\mu_0 I}{2\pi d} \\ d \ll L \quad &\Rightarrow \quad B = \frac{\mu_0 I}{2L} \overset{d}{=} \frac{\mu_0}{2} j \end{aligned}$$

Cross the plane: $\Delta B = \mu_0 j$.

Circular wire

Radius: R, distance to center: x, to wire: r.

$$\begin{aligned} B &= \oint \frac{\mu_0}{4\pi} \frac{I \,\mathrm{d}l}{r^2} \sin\theta = \frac{\mu_0 I R}{2r^2} \frac{R}{r} \\ &= \frac{\mu_0 I R^2}{2r^3} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \end{aligned}$$

If $x = 0$, then $B = \dfrac{\mu_0 I}{2R}$.

For N turn loops, $B = N \dfrac{\mu_0 I}{2R}$.

A Stretched-out Solenoid

Choose $\mathrm{d}I = nI\,\mathrm{d}x$, $x = R\cot\beta$.

$$\begin{aligned} B &= \int_{x_1}^{x_2} \frac{\mu_0 n I R^2\,\mathrm{d}x}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 n I}{2}\int_{\beta_1}^{\beta_2}(-\sin\beta)\,\mathrm{d}\beta \\ &= \frac{\mu_0 n I}{2}(\cos\beta_2 - \cos\beta_1) \end{aligned}$$

To infinity: $B = \mu_0 n I$, where n stands for number density.

Ampere's Circuital Theorem

$$\oint_L \vec{\boldsymbol{B}}\cdot\mathrm{d}\vec{\boldsymbol{l}} = \mu_0\sum_i I_i$$

Toroid

Toroid is not solenoid!

$$B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I$$

Cylindrical conductor

$$B = \begin{cases} \dfrac{\mu_0 I}{2\pi r} & (r > R) \\ \dfrac{\mu_0 I r}{2\pi R^2} & (r < R) \end{cases}$$

Magnetic Field Due to Moving Charges

$$\begin{aligned} I\,\mathrm{d}\vec{\boldsymbol{l}} &= q\vec{\boldsymbol{v}}\,ns\,\mathrm{d}l = q\vec{\boldsymbol{v}}\,\mathrm{d}N \\ \mathrm{d}\vec{\boldsymbol{B}} &= \frac{\mu_0}{4\pi}\frac{I}{r^2}\,\mathrm{d}\vec{\boldsymbol{l}}\times\hat{\boldsymbol{r}} \\ \vec{\boldsymbol{B}} &= \frac{\mathrm{d}\vec{\boldsymbol{B}}}{\mathrm{d}N} = \frac{\mu_0}{4\pi}\frac{q\,\vec{\boldsymbol{v}}\times\hat{\boldsymbol{r}}}{r^2} \end{aligned}$$

Magnetic Field

Defined by **Lorenz force**.

$$\begin{aligned} \vec{\boldsymbol{F}} &\equiv q\,\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}} \\ \vec{\boldsymbol{\tau}} &\equiv NIS\,\vec{\boldsymbol{n}}\times\vec{\boldsymbol{B}} \end{aligned}$$

Magnetic Flux

Defined by analogy with electric flux, with unit of Wb.

$$\begin{aligned} \Phi_m &= \iint_S \vec{\boldsymbol{B}}\cdot\mathrm{d}\vec{\boldsymbol{A}} \\ \Phi_m &= \oiint_S \vec{\boldsymbol{B}}\cdot\mathrm{d}\vec{\boldsymbol{A}} \equiv 0 \end{aligned}$$

Monopoles doesn't exist.

Mass spectrograph.

The Hall Effect

P/N type semiconductor.

$$I = nedbv$$
$$V = Eb = vBb = \frac{BI}{ned} \equiv \frac{BIR}{d}$$

R: Hall coefficient.

The cyclotron accelerator.

Magnetic Force On Current

$$\vec{\mathbf{F}} = q\,\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}} = \int I\,\mathrm{d}\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}}$$

Magnetic Moment

$$\vec{\boldsymbol{m}} \equiv \vec{\boldsymbol{\mu}} \equiv NI\vec{\boldsymbol{A}}$$

Torque on a Current Loop

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}$$

Galvanometer

The Magnetic Dipole Moment

$$W = \int_{\theta_1}^{\theta_2} \tau\,\mathrm{d}\theta = \left(-\mu B\cos\theta\right|_{\theta_1}^{\theta_2}$$
$$U(\theta) = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$$

Chapter 11 Electronmagnetic Induction

11.1 EMI Phenomena

11.2 Faraday's law and Lenz's Law

11.3 Induced Electric Field

11.3 Self-Inductance and Mutual-Inductance

11.5 The Energy Stored in Magnetic Field

11.6 Applications

Motional electromotive force (emf) 感应电动势

11. 稳恒电流和运动电荷的磁场

11.1 恒定电流

- 电流分类
    1. 传导电流：导体中带电粒子定向运动
    2. 运流电流：带电物体机械运动
    3. 位移电流：变化的电场
- 电流元：$I\,\mathrm{d}\vec{l}$
- 宏观与微观：$I=\dfrac{\mathrm{d}q}{\mathrm{d}t}=nqsv$
- 电流密度：$\vec{j}\overset{d}{=}\dfrac{\mathrm{d}I}{\mathrm{d}S_{\perp}}\vec{e}_v$
- 恒定电流连续性方程：$\oiint\limits_S \vec{j}\cdot\mathrm{d}\vec{s}=-\dfrac{\mathrm{d}q}{\mathrm{d}t}$
- 基尔霍夫定律（节点定律）
- 电动势：$\varepsilon=\dfrac{W}{q}=\oint\vec{E}_k\cdot\mathrm{d}\vec{l}$
    - 开源时路端电压等于电源电动势
    - 电动势与路径有关

11.2 磁场、磁感应强度

- 磁矩：$\vec{m}=IS\vec{e}_n$
    - 分子电流相当于小磁体
    - 其中 $\vec{e}_n$ 与 I 成右手螺旋
- 磁感应强度
    1. 小磁针
    2. 运动电荷
        - $\vec{F}\equiv q\vec{v}\times\vec{B}$
    3. 载流线圈：稳定平衡时 $\vec{m}$ 的方向
        - $\vec{\tau}\equiv NIS\vec{n}\times\vec{B}$

11.3 毕奥萨瓦定律

11.4 运动电荷产生磁场

- 单个运动电荷 ($v \ll c$)
  - $I\,\mathrm{d}\vec{\boldsymbol{l}} = q\vec{\boldsymbol{v}}\,ns\,\mathrm{d}l = q\vec{\boldsymbol{v}}\,\mathrm{d}N$
  - $\mathrm{d}\vec{\boldsymbol{B}} = \dfrac{\mu_0}{4\pi}\dfrac{I}{r^2}\vec{\boldsymbol{l}} \times \hat{\boldsymbol{r}}$
  - $\vec{\boldsymbol{B}} = \dfrac{\mathrm{d}\vec{\boldsymbol{B}}}{\mathrm{d}N} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{\boldsymbol{v}} \times \hat{\boldsymbol{r}}}{r^2}$
- 电子圆周运动：$B_0 = \dfrac{\mu_0 ev}{4\pi r^2}$
- 闭合电流磁矩：$m = IS = \dfrac{ev}{2\pi r}\pi r^2 = \dfrac{evr}{2}$
- 旋转带电圆盘：
  - $\mathrm{d}I = \dfrac{\mathrm{d}q}{T} = \dfrac{\sigma \cdot 2\pi r\,\mathrm{d}r}{2\pi/\omega} = \omega\sigma r\,\mathrm{d}r$
  - $\mathrm{d}B = \dfrac{\mu_0\,\mathrm{d}I}{2r} = \dfrac{\mu_0\omega\sigma\,\mathrm{d}r}{2}$, $B = \dfrac{\mu_0\omega\sigma R}{2}$
  - $\mathrm{d}m = s\,\mathrm{d}I = \pi r^2 \cdot \omega\sigma r\,\mathrm{d}r$, $m = \dfrac{\pi\omega\sigma R^4}{4}$

12. 运动电荷、载流导线受力

12.1 磁力线、磁通量、磁场的高斯定理

- 磁通量：
  - $\Phi = \vec{\boldsymbol{B}}\boldsymbol{\cdot}\vec{\boldsymbol{S}}.$
  - 穿出为正

12.2 安培环路定理

$$\oint_L \vec{\boldsymbol{B}}\boldsymbol{\cdot}\,\mathrm{d}\vec{\boldsymbol{l}} = \mu_0 \sum I_{\text{内}}$$

> 磁场是非保守场

12.3 洛伦兹力

洛伦兹力：$F = q\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}$

- 匀强磁场
  - 磁聚焦
  - 磁发散
- 非匀强磁场
  - 磁镜效应
    - 横向磁约束
    - 纵向磁约束
  - 磁约束

12.4 应用

- 回旋加速器
- 质谱仪
- 霍尔效应
  - 测量半导体特征
  - 霍尔传感器
  - 磁流体船

12.5 安培定律

$$\vec{\boldsymbol{F}}=\int I\,\mathrm{d}\vec{\boldsymbol{l}}\times\vec{\boldsymbol{B}}$$

无限长平行直导线：$\dfrac{\mathrm{d}F_{21}}{\mathrm{d}l_2}=\dfrac{\mu_0 I_1 I_2}{2\pi a}$.

13 电磁感应（1）

13. 1 磁介质、顺磁质、抗磁质磁化

磁介质：经磁化后能够影响磁场分布的物质。

- 传导电流：$I_0\to\vec{\boldsymbol{B}}_{\mathbf{0}}$
- 介质磁化：$\to\vec{\boldsymbol{B}}'_{\mathbf{0}}$
- 总磁感应强度：$\vec{\boldsymbol{B}}=\vec{\boldsymbol{B}}_{\mathbf{0}}+\vec{\boldsymbol{B}}'$
- 相对磁导率：$\vec{\boldsymbol{B}}=\mu_r\vec{\boldsymbol{B}}_{\mathbf{0}}$

> 均匀各向同性介质充满磁场所在空间

分类：

- 顺磁质（Paramagnetic substance），弱磁质：Mn, Al, O_2
  - $\vec{\boldsymbol{B}}'\uparrow\vec{\boldsymbol{B}}_{\mathbf{0}}\uparrow$，且 $\vec{\boldsymbol{B}}'\ll\vec{\boldsymbol{B}}_{\mathbf{0}},\mu_r>1.$
  - 分子具有固定的分子磁矩（电子轨道、自旋磁矩）
  - 磁场下表面表现出束缚（磁化）电流，加强磁场
- 抗磁质（Diamagnetic substance），弱磁质：Cu, Ag, H_2
  - $\vec{\boldsymbol{B}}'\uparrow\vec{\boldsymbol{B}}_{\mathbf{0}}\downarrow$，且 $\vec{\boldsymbol{B}}'\ll\vec{\boldsymbol{B}}_{\mathbf{0}},\mu_r<1.$
  - 无论电子绕原子核转动方向为何，其磁矩在 $\vec{\boldsymbol{B}}$ 方向投影均小于零.
- 铁磁质（Ferromagnetic substance）强磁质：Fe, Co, Ni
  - $\vec{\boldsymbol{B}}'\uparrow\vec{\boldsymbol{B}}_{\mathbf{0}}\uparrow$，且 $\vec{\boldsymbol{B}}'\gg\vec{\boldsymbol{B}}_{\mathbf{0}},\mu_r\gg1.$
  - 非线性关系，且非单值对应关系.

13.2 磁介质中的安培环路定理

磁化电流与传导电流

- 传导电流：有热效应
- 磁化电流：无热效应
- 磁化面电流密度：$j_s \equiv \dfrac{I_s}{l}$.
- 磁化强度：$\vec{\boldsymbol{M}} \equiv \dfrac{\sum \vec{\boldsymbol{m}}_{分}}{\Delta V}$，$\left|\vec{\boldsymbol{M}}\right| = \dfrac{I_s S}{lS} = j_s$.
- 积分关系：$\displaystyle\oint_L \vec{\boldsymbol{M}} \cdot \mathrm{d}\vec{\boldsymbol{l}} = Ml = I_s$.

安培环路定理

- 真空中：$\displaystyle\oint_L \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{l}} = \mu_0 \sum I_i$.
- 介质中：$\displaystyle\oint_L \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{l}} = \mu_0 \sum (I_C + I_S) = \mu_0 \sum I_C + \mu_0 \oint_L \vec{\boldsymbol{M}} \cdot \mathrm{d}\vec{\boldsymbol{l}}$.
- 磁场强度：$\vec{\boldsymbol{H}} \equiv \dfrac{\vec{\boldsymbol{B}}}{\mu_0} - \vec{\boldsymbol{M}}$.
- 介质中：$\displaystyle\oint_L \vec{\boldsymbol{H}} \cdot \mathrm{d}\vec{\boldsymbol{l}} = \sum I_C$.

> 仅适用于稳恒情况

- 各向同性介质：$\vec{\boldsymbol{M}} = \kappa \vec{\boldsymbol{H}}$，磁化率：$\kappa$.
  - $\vec{\boldsymbol{B}} = \mu_0(1+\kappa)\vec{\boldsymbol{H}} = \mu_0\mu_r\vec{\boldsymbol{H}}$.
  - 相对磁导率：$\mu_r = 1 + \kappa$.
  - 磁导率：$\mu = \mu_0\mu_r$.
  - 螺绕圈：$B = \mu H = \mu n I_0$.

13.3 法拉第电磁感应定律

楞次定律

法拉第电磁感应定律

- 感应电动势：$\varepsilon = -\dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = \dfrac{B\,\mathrm{d}S}{\mathrm{d}t}\cos\theta$.
  - 动生电动势、感生电动势
- 感应电荷：$\displaystyle q = \int_{t_1}^{t_2} I\,\mathrm{d}t = \int_{t_1}^{t_2} -\frac{1}{R}\frac{\mathrm{d}\Phi}{\mathrm{d}t}\,\mathrm{d}t = \frac{\Phi_1 - \Phi_2}{R}$.
  - 磁通计

13.4 动生电动势

普遍表达式

$$\mathrm{d}\Phi = \vec{\boldsymbol{B}} \cdot \mathrm{d}\vec{\boldsymbol{s}} = \vec{\boldsymbol{B}} \cdot (\vec{\boldsymbol{v}} \times \mathrm{d}\vec{\boldsymbol{l}})\,\mathrm{d}t$$

$$\mathrm{d}\varepsilon = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\vec{\boldsymbol{B}} \cdot (\vec{\boldsymbol{v}} \times \mathrm{d}\vec{\boldsymbol{l}}) = (\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}}) \cdot \mathrm{d}\vec{\boldsymbol{l}}$$

$$\varepsilon_{ab}=\int_a^b(\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}})\cdot\mathrm{d}\vec{\boldsymbol{l}}$$

微观本质

$$\vec{\boldsymbol{E}}_k=\frac{\vec{\boldsymbol{F}}_k}{-e}=\vec{\boldsymbol{v}}\times\vec{\boldsymbol{B}}$$

14 电磁感应（2）

14.1 感生电动势

1. 麦克斯韦第一假设

不论有无导体或回路，变化的磁场在周围空间中产生一种感生电场（涡旋电场，或称右旋电场），这种电场的电力线是闭合曲线，其线积分即为感生电动势。

$$\varepsilon=\oint_L\vec{\boldsymbol{E}}_v\cdot\mathrm{d}\vec{\boldsymbol{l}}=-\iint_S\frac{\partial\vec{\boldsymbol{B}}}{\partial t}\cdot\mathrm{d}\vec{\boldsymbol{s}}$$

2. 说明

而数学上若 $\oint_L\vec{\boldsymbol{a}}\cdot\mathrm{d}\vec{\boldsymbol{l}}=\iint_S\vec{\boldsymbol{b}}\cdot\mathrm{d}\vec{\boldsymbol{s}}$，则 $\vec{\boldsymbol{a}}$ 与 $\vec{\boldsymbol{b}}$ 成右手螺旋关系，

故 $\vec{\boldsymbol{E}}_v$ 与 $\dfrac{\partial\vec{\boldsymbol{B}}}{\partial t}$ 成左螺旋关系。

3. 应用

- 电子感应加速器
- 涡电流
  - 热效应
    - 电磁炉、高频驱动
  - 机械效应
    - 电磁阻尼、电磁驱动、电磁式转速表

14.2 自感和互感

1. 自感系数

定义

$L=\dfrac{\varPsi}{I}$，单位：H（亨利），$\varPsi$ 为磁通量

影响因素

- 与几何因素、磁介质有关
- 无铁磁质时，与 I 无关

2. 自感电动势

$\varepsilon_L = -\frac{\mathrm{d}\Psi_m}{\mathrm{d}t} = -\frac{\mathrm{d}(IL)}{\mathrm{d}t} = -L\frac{\mathrm{d}I}{\mathrm{d}t}$

3. 长直密螺绕线管

$$
\begin{aligned}
B &= \mu H = \mu nI = \frac{\mu NI}{l} \\
\Psi &= N\Phi = NBS = \mu\frac{N^2}{l}SI \\
L &= \frac{\Psi}{I} = \mu\frac{N^2}{l}S = \mu n^2 V
\end{aligned}
$$

4. 互感系数

定义

$M_{21} = \frac{\Psi_{21}}{I_1}$

$\varepsilon_{21} = -\frac{\mathrm{d}\Psi_{21}}{\mathrm{d}t} = -M_{21}\frac{\mathrm{d}I_1}{\mathrm{d}t}$

$M_{21} = M_{12} = M$

影响因素

只与线圈的形状、大小、匝数、相对位置、周围磁介质的磁导率有关

5. 两线圈缠绕在一起

$$
\begin{aligned}
\Psi_{21} &= N_2\Phi_{21} = N_2B_1S = N_2\mu n_1 I_1 S \\
M &= \frac{\Psi_{21}}{I_1} = \mu n_1 N_2 S = \mu n_1 n_2 V \\
M &= \sqrt{L_1 L_2}
\end{aligned}
$$

一般情况下有：$M = k\sqrt{L_1L_2}$，其中 $k \in [0,1]$.

6. 应用

- 自感
  - 日光灯中的镇流器
- 互感
  - 变压器、信号传递

14.3 磁场能量

1. 磁场能量的自感表达

$$
\begin{aligned}
\mathrm{d}W = \varepsilon\,\mathrm{d}q = -L\frac{\mathrm{d}i}{\mathrm{d}t}\,\mathrm{d}q = -Li\,\mathrm{d}i \\
W = \int_I^0 -Li\,\mathrm{d}i = \frac{1}{2}LI^2
\end{aligned}
$$

2. 磁场能量密度的表述

以长直螺线管为例，但对一般情况亦适用。

$$
\begin{aligned}
L &= \mu n^2 V \\
B &= \mu H = \mu n I \\
W_m &= \frac{1}{2}LI^2 = \frac{B^2V}{2\mu} \\
w_m &= \frac{B^2}{2\mu} = \frac{1}{2}\mu H^2 = \frac{1}{2}BH \\
W_m &= \iiint_V w_m\,\mathrm{d}v = \iiint_V \frac{B^2}{2\mu}\,\mathrm{d}v
\end{aligned}
$$

14.4 位移电流、麦克斯韦方程组

一、位移电流

稳恒磁场：$\oint_L \vec{\boldsymbol{H}}\cdot\mathrm{d}\vec{\boldsymbol{l}} = \sum I.$

非稳恒磁场：

极板上电荷面密度 σ，电位移矢量 $D(t)$ 的通量为 ϕ_D.

$\phi_D(t) = D(t)S = \sigma S = q$

位移电流：$I_d \equiv \dfrac{\mathrm{d}\phi_D}{\mathrm{d}t}.$

位移电流密度：$\vec{\boldsymbol{j}}_d \equiv \dfrac{\partial\vec{\boldsymbol{D}}}{\partial t}.$

$I_d = \dfrac{\mathrm{d}}{\mathrm{d}t}\iint_S \vec{\boldsymbol{D}}\cdot\mathrm{d}\vec{\boldsymbol{S}} = \iint_S \dfrac{\partial\vec{\boldsymbol{D}}}{\partial t}\cdot\mathrm{d}\vec{\boldsymbol{S}} = \iint_S \vec{\boldsymbol{j}}_d\cdot\mathrm{d}\vec{\boldsymbol{S}}$

位移电流 *不产生* 焦耳热。

二、全电流的安培环路定理

全电流：$I = I_c + I_d$

$\oint_L \vec{\boldsymbol{H}}\cdot\mathrm{d}\vec{\boldsymbol{l}} = \sum I = \sum I_c + \dfrac{\mathrm{d}\phi_d}{\mathrm{d}t}$

应用

三、麦克斯韦方程组

定理整理

1. 静电场高斯定理：$\oint_S \vec{\boldsymbol{D}}\cdot \mathrm{d}\vec{\boldsymbol{S}} = \sum q_f = \iiint_V \rho\, \mathrm{d}V$
2. 静电场环路定理：$\oint_L \vec{\boldsymbol{E}}\cdot \mathrm{d}\vec{\boldsymbol{l}} = 0$
3. 磁场高斯定理：$\oiint_S \vec{\boldsymbol{B}}\cdot \mathrm{d}\vec{\boldsymbol{S}} = 0$
4. 安培环路定理：$\oint_L \vec{\boldsymbol{H}}\cdot \mathrm{d}\vec{\boldsymbol{l}} = \sum I = \iint_S \vec{\boldsymbol{j}}\cdot \mathrm{d}\vec{\boldsymbol{s}}$

麦克斯韦假设

1. 有旋电流：$\oint_L \vec{\boldsymbol{E}}_v\cdot \mathrm{d}\vec{\boldsymbol{l}} = -\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = -\iint_S \frac{\partial \vec{\boldsymbol{B}}}{\partial t}\cdot \mathrm{d}\vec{\boldsymbol{S}}$
2. 位移电流：$I_d = \frac{\mathrm{d}\phi_d}{\mathrm{d}t} = \iint_S \vec{\boldsymbol{j}}_d\cdot \mathrm{d}\vec{\boldsymbol{S}}$

麦克斯韦方程组（积分形式）

$$\begin{cases} \oiint_S \vec{\boldsymbol{D}}\cdot \mathrm{d}\vec{\boldsymbol{S}} = \sum q_f \\ \oint_L \vec{\boldsymbol{E}}\cdot \mathrm{d}\vec{\boldsymbol{l}} = -\iint_S \frac{\partial \vec{\boldsymbol{B}}}{\partial t}\cdot \mathrm{d}\vec{\boldsymbol{S}} \\ \oiint_S \vec{\boldsymbol{B}}\cdot \mathrm{d}\vec{\boldsymbol{S}} = 0 \\ \oint_L \vec{\boldsymbol{H}}\cdot \mathrm{d}\vec{\boldsymbol{l}} = \iint_S (\vec{\boldsymbol{j}}_c + \frac{\partial \vec{\boldsymbol{D}}}{\partial t})\cdot \mathrm{d}\vec{\boldsymbol{S}} \end{cases}$$

麦克斯韦方程组（微分形式）

1. 高斯定理

$$\nabla\cdot\vec{\boldsymbol{D}} = \rho_0$$
$$\nabla\cdot\vec{\boldsymbol{B}} = 0$$

2. 斯托克斯公式

$$\nabla\times\vec{\boldsymbol{E}}_{静} = 0$$
$$\nabla\times\vec{\boldsymbol{E}} = -\frac{\partial \vec{\boldsymbol{B}}}{\partial t}$$
$$\nabla\times\vec{\boldsymbol{H}} = \vec{\boldsymbol{j}}_c + \frac{\partial \vec{\boldsymbol{D}}}{\partial t}$$

四、应用

$$\left.\begin{array}{r} \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial x^2} = \varepsilon\mu \frac{\partial^2 \vec{\boldsymbol{E}}}{\partial t^2} \\ \frac{\partial^2 \vec{\boldsymbol{H}}}{\partial x^2} = \varepsilon\mu \frac{\partial^2 \vec{\boldsymbol{H}}}{\partial t^2} \\ \frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 y}{\partial t^2} \end{array}\right\} \quad \Rightarrow \quad u = \frac{1}{\sqrt{\varepsilon\mu}} = c$$